

IN-39

NASA Technical Memorandum 104367

11691

P14

Analysis of Some Compliance Calibration Data for Chevron-Notch Bar and Rod Specimens

(NASA-TM-104367) ANALYSIS OF SOME
COMPLIANCE CALIBRATION DATA FOR
CHEVRON-NOTCH BAR AND ROD SPECIMENS (NASA)

14 p

CSCL 20K

N91-22594

Unclass

63/39 0011691

Thomas W. Orange, Raymond T. Bubsey, William S. Pierce,
and John L. Shannon, Jr.
Lewis Research Center
Cleveland, Ohio

Prepared for the
Symposium on Chevron-Notch Fracture Test Experience
sponsored by the American Society for Testing and Materials
Indianapolis, Indiana, May 6, 1991

NASA



ANALYSIS OF SOME COMPLIANCE CALIBRATION DATA FOR CHEVRON-NOTCH BAR AND ROD SPECIMENS

Thomas W. Orange, Raymond T. Bubsey,* William S. Pierce,* and John L. Shannon, Jr.
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

This paper presents a set of equations describing certain fracture mechanics parameters for chevron-notch bar and rod specimens. They are developed by fitting earlier compliance calibration data. The difficulty in determining the minimum stress intensity coefficient and the critical crack length is discussed.

INTRODUCTION

This paper presents a set of equations describing certain fracture mechanics parameters for chevron-notch bar and rod specimens. They are developed by fitting previously reported experimental compliance calibration data. Their use will facilitate the testing and analysis of both brittle metals and the tougher ceramics. The equations present the various parameters in forms suitable for determining fracture toughness from maximum load, for determining the crack-extension resistance curve (R-curve), and for setting instrument sensitivities. The data encompass the entire range of the specimen geometries most commonly used.

We first discuss briefly the background of the chevron-notch specimens and the experimental data to be used. Then we present a more extensive discussion on some particular characteristics of the chevron-notch specimens and their practical application. The fitted equations are presented and their fitting accuracies are discussed. Finally, problems in determining the minimum stress intensity coefficient and the critical crack length are discussed.

SYMBOLS

a	Crack length (measured from load line)
a_0	Distance from load line to tip of chevron
a_m	Crack length at which Y is minimum
B	Specimen thickness
C	Specimen compliance, $C = EBV/P$
C'	Compliance derivative, $dC/d\alpha$
D	Diameter (rod), $D = B$

*Retired.

E	Elastic (Young's) modulus
K_I	Opening-mode stress intensity factor
K_N	Plane-strain fracture toughness for chevron-notch specimens
P	Applied load
V	Crack mouth opening displacement
W	Width
Y	Dimensionless stress intensity factor for a crack in a chevron notch, $K_I B W^{1/2} / P$
Y_m	Minimum value of Y as a function of a
α	a/W
α_0	a_0/W
α_m	a_m/W

BACKGROUND

The chevron-notch specimens are fairly recent additions to the field of fracture mechanics. Consequently they do not have the same historical background of extensive stress intensity and displacement analysis as do the more common specimen types. But, like the earliest specimen types, we can develop useful expressions using experimental compliance data.

Compliance data for the chevron-notch bar [1] and rod [2] specimens were previously reported. In each paper, one fitted equation was presented relating the minimum stress intensity factor to the initial crack length and to the specimen dimensions. A later paper [3] reported additional data for specimens having smaller initial crack lengths and also revised the previous equations to cover the wider range of crack lengths. But those equations alone are not sufficient for all analyses and tests involving high-toughness ceramics. To make them more complete and useful, a new set of generalized equations are presented in this paper. These equations are developed by fitting curves to the existing data. They are usable over a wide range of specimen dimensions.

CHARACTERISTICS OF CHEVRON-NOTCH SPECIMENS

For most common fracture test specimens, the dimensionless stress intensity factor (Y) increases continually with increasing relative crack length (a/W). But due to the wedge shape of the un-notched material in the chevron-notch specimen, the corresponding factor (Y) reaches a minimum, denoted Y_m , as the crack length reaches a value denoted a_m . The values of Y_m and a_m are functions of specimen dimensions and notch geometry only and are independent of material properties.

If the material being tested has a crack growth resistance curve which increases rapidly to a relatively constant plateau (known as a "flat" R-curve), instability will occur at $a=a_m$ and $P=P_{max}$. Then the fracture toughness (K_N) can be calculated from

$$K_{Iv} = Y_m^* \frac{P_{max}}{BW^{1/2}} \quad (1)$$

and no other test measurements are necessary.

For some materials (even some ceramics), however, the R-curve does not reach a plateau but continues to increase with increasing crack extension (a "rising" R-curve). For such materials eqn. (1) does not apply and it may be desirable to determine the complete R-curve. In this case ASTM Test Method E 561 [4] may be used for guidance. If crack mouth opening displacement (CMOD) is measured during the test (as in E561) and appropriate compliance relations are available, one can calculate the instantaneous crack length. From crack length and load, one can calculate the crack extension resistance as

$$K_I = Y^* \frac{P}{BW^{1/2}} \quad (2)$$

A plot of crack extension resistance against crack advance is the R-curve.

PROCEDURE

Experimental

The experimental procedure is described in detail in Refs [1,2]. The complete data are presented in Ref [5]. At least three replicate tests for each crack length were averaged to obtain the data reported here. For each specimen, 7 to 15 crack lengths (depending on the initial crack length) were tested.

Basic data reduction

Analysis of the data is based on the following equation [1]

$$Y^* = \left[\frac{1}{2} \frac{\alpha_1 - \alpha_0}{\alpha - \alpha_0} \frac{d}{d\alpha} \frac{EBV}{P} \right]^{1/2} \quad (3)$$

and its derivative with respect to α . In [1-3] the logarithms of the basic compliance data ($C = EBV/P$) were fit with a fourth degree polynomial in α . The fitted curve was differentiated and the values of Y^* calculated from Eq 3.

In Ref [1] the reported values of Y_m^* and α_m corresponded to the minimum of that fitted curve. In Ref [2], Y_m^* and α_m were determined in the same way, but the data range was restricted to seven points symmetrical about the value of α_m found by the first fitting. Ref [3] used still another procedure. Seven points were selected by the previous criterion. Then a fourth degree polynomial was fit to the logarithms of the

compliance derivatives. That second polynomial was used to calculate Y_m^* and α_m .

In the process of verifying these calculations, some general concerns arose concerning procedures for determining Y_m^* and α_m . These will be discussed later.

Development of generalized equations

The following expressions are useful for computing the plane strain fracture toughness K_{Ic} when the material has a relatively "flat" R-curve.

$$\alpha_m = A_0 + A_1 \alpha_0 + A_2 \alpha_0^2 + A_3 \alpha_0^3 \quad (4)$$

and

$$Y_m^* = B_0 + B_1 \alpha_0 + B_2 \alpha_0^2 + B_3 \alpha_0^3 \quad (5)$$

These were developed by first fitting third-degree polynomials in α_0 for each specimen type (bar or rod) and each value of W/B . Then the coefficients of the intermediate polynomials were in turn fit to a second-degree polynomial in W/B to produce the final forms of Eqns (4) and (5). Values of the coefficients for Eqns (4) and (5) are given in Tables 1 and 2, respectively.

An expression for determining the relative crack length α as a function of measured displacements is

$$\alpha = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 \quad (6)$$

where U is the Saxena and Hudak form [6]

$$U = \frac{1}{\left(\frac{EBV}{P}\right)^{1/2} + 1}$$

The coefficients for Eqn (6) are given in Table 3. This equation lends itself to computer-controlled fracture toughness testing since the subcritical crack growth can be determined from automated load and displacement data acquisition.

When the relative crack length α is known, the stress intensity factor Y^* and the dimensionless compliance EBV/P can be computed from the following expressions:

$$Y^* = e^{D_0 + D_1 \alpha + D_2 \alpha^2 + D_3 \alpha^3 + D_4 \alpha^4} \quad (7)$$

and

$$\frac{EBV}{P} = e^{E_0 + E_1 \alpha + E_2 \alpha^2 + E_3 \alpha^3 + E_4 \alpha^4} \quad (8)$$

The coefficients for Eqns (7) and (8) are given in Tables 4 and 5, respectively.

DISCUSSION

Generalized equations

Eqn (4) fits the calculated values of α_m within 0.013W for the bar specimens and within 0.006W for the rod specimens. Eqn (5) fits the calculated values of Y_m^* within 1.0% for the bar specimens and within 2.7% for the rod specimens.

Within the ranges of α and α_0 specified in Tables 3 to 5, Eqn (6) fits the measured values of α within 0.003W for the bar specimen and within 0.002W for the rod specimen; Eqn (7) fits the calculated values of Y^* within 2.9% for the bar specimen and within 2.1% for the rod specimen; and Eqn (8) fits the measured values of EBV/P within 1.4% for both the bar and the rod specimen.

Table 3 of Ref [7] gives values of Y_m^* and a critical slope ratio r_c . That ratio is the ratio of the compliances corresponding to α_m and α_0 . For specimens with $W/B=2.0$, the values of Y_m^* computed from Eq (5) for both the bar and rod specimens are within 0.6% of those in Ref [7]. The critical slope ratio computed from Eq (8) is within 1% for the bar specimen but is 7.8% low for the rod specimen.

Problems in determining Y_m^* and α_m

The method of data analysis used in Ref [3] was not given explicitly and could not be determined directly from archival records. In attempting to verify the numerical analysis (by duplication), several methods were tried. Each produced a significantly different value for α_m for the same data set, and this is a problem that should be discussed.

The problem is inherent in the chevron-notch specimen. It is due to the same characteristic that makes it desirable, namely the fact that Y^* has a minimum. For example, assume that we have a function f such that

$$EBV/P = f(\alpha)$$

where f includes the data transform (if any) and a fitting function. Substituting this into the derivative of Eq (3) and eliminating non-zero terms we have

$$0 = \frac{1}{\alpha_m - \alpha_0} f'(\alpha_m) - f''(\alpha_m) \quad (9)$$

where f' and f'' are the first and second derivatives and α_m is the root of this equation.

Unlike simpler specimens, we need to determine the second derivative as well. This presents a strong challenge to the analyst.

Fig 2, from Ref [1], shows the typical variation of Y^* with α for different values of α_0 . Experimental compliance data would be expected to scatter about these lines. It is apparent from this figure that for a short initial crack (say, $\alpha_0=0.2$) Y_m^* will be relatively insensitive to the method of curve fitting but α_m will be very sensitive. However, for a long initial crack (say, $\alpha_0=0.5$) the opposite will be true.

Thus if the primary objective of the test is to determine K_m , the initial crack length should be short. This is the case in Ref [7]. However, a long initial crack length is preferable if the critical crack length is important for, say, fractographic purposes.

It should be pointed out that numerical analyses (i.e., finite element or boundary integral methods) are subject to the same problem, although to a lesser degree. Discrete pairs of (Y^* , α_0) for several initial crack lengths must be fitted with a function to calculate a minimum. Three pairs are required, more would be preferred.

CONCLUSIONS

The equations presented here are in forms suitable for several purposes in fracture testing with chevron-notch specimens. They encompass the range of specimen geometries most commonly used and provide a good fit with the basic compliance data. The inherent difficulty in determining the critical crack length from compliance measurements is discussed.

REFERENCES

- [1] Munz, D., Bubsey, R. T., and Srawley, J. E., "Compliance and Stress Intensity Coefficients for Short Bar Specimens with Chevron Notches," *International Journal of Fracture*, Vol. 16, No. 4, August 1980, pp. 359-374.
- [2] Bubsey, R. T., Munz, D., Pierce, W. S., and Shannon, J. L., Jr, "Compliance Calibration of the Short Rod Chevron-Notch Specimen for Fracture Toughness Testing of Brittle Materials," *International Journal of Fracture*, Vol. 18, No. 2, February 1982, pp. 125-133.
- [3] Shannon, J. L., Jr., Bubsey, R. T., Pierce, W. S., and Munz, D., "Extended Range Stress Intensity Factor Expressions for Chevron-Notched Short Bar and Short Rod Fracture Toughness Specimens," *International Journal of Fracture*, Vol. 19, No. 3, July 1982, pp. R55-R58.

- [4] Standard Practice for R-Curve Determination, ASTM E 561-86, *Annual Book of ASTM Standards*, Vol. 03.01, 1990.
- [5] Bubsey, R. T., Orange, T. W., Pierce, W. S., and Shannon, J. L., Jr., "Closed-Form Expressions for Crack-Mouth Displacements and Stress Intensity Factors for Chevron-Notched Short Bar and Short Rod Specimens Based on Experimental Compliance Measurements," *NASA TM 83796*, 1991.
- [6] Saxena, A., and Hudak, S. J., Jr., "Review and Extension of Compliance Information for Common Crack Growth Specimens," *International Journal of Fracture*, Vol. 14, No. 5, October 1978, pp. 453-468.
- [7] Standard Test Method for Plane-Strain (Chevron-Notch) Fracture Toughness of Metallic Materials, ASTM E 1304-89, *Annual Book of ASTM Standards*, Vol. 03.01, 1990.

Table 1--Coefficients for Eq 1

Specimen	Coeff.	Expression
Bar	A_0	$-0.110 + 0.354(W/B) - 0.088(W/B)^2$
	A_1	$0.268 + 1.628(W/B) - 0.400(W/B)^2$
	A_2	$1.637 - 6.358(W/B) + 1.872(W/B)^2$
	A_3	$0.075 + 4.462(W/B) - 1.508(W/B)^2$
Rod	A_0	$0.147 + 0.089(W/B) - 0.026(W/B)^2$
	A_1	$0.358 + 1.150(W/B) - 0.096(W/B)^2$
	A_2	$2.860 - 5.190(W/B) + 0.770(W/B)^2$
	A_3	$-3.610 + 5.100(W/B) - 0.800(W/B)^2$

Range: $1.5 \leq (W/B) \leq 2.0$, $0 \leq \alpha_0 \leq 0.5$

Table 2--Coefficients for Eq 5

Specimen	Coeff.	Expression
Bar	B_0	$-17.03 + 29.94(W/B) - 5.0(W/B)^2$
	B_1	$-116.00 + 141.60(W/B) - 29.6(W/B)^2$
	B_2	$1131.00 - 1304.00(W/B) + 342.0(W/B)^2$
	B_3	$-1351.00 + 1654.00(W/B) - 443.2(W/B)^2$
Rod	B_0	$5.47 + 6.29(W/B) + 2.46(W/B)^2$
	B_1	$-65.93 + 72.62(W/B) - 5.62(W/B)^2$
	B_2	$622.00 - 659.80(W/B) + 146.10(W/B)^2$
	B_3	$-541.40 + 629.10(W/B) - 135.20(W/B)^2$

Range: $1.5 \leq (W/B) \leq 2.0$, $0 \leq \alpha_0 \leq 0.5$

Table 3--Coefficients for Eq 6

Specimen	Coeff.	Expression
Bar W/B=1.5	C_0	3.09 $-24.12\alpha_0 +57.12\alpha_0^2$
	C_1	-109.30 $+1227.00\alpha_0 -2876.00\alpha_0^2$
	C_2	1 908.00 $-22216.00\alpha_0 +51286.00\alpha_0^2$
	C_3	-14 900.00 $+168580.00\alpha_0 -381240.00\alpha_0^2$
	C_4	41 390.00 $-451059.00\alpha_0 +987080.00\alpha_0^2$
Bar W/B=2.0	C_0	2.08 $-8.74\alpha_0 +16.93\alpha_0^2$
	C_1	-63.31 $+540.00\alpha_0 -1019.00\alpha_0^2$
	C_2	1 086.00 $-11296.00\alpha_0 +20043.00\alpha_0^2$
	C_3	-9 327.00 $+98493.00\alpha_0 -158690.00\alpha_0^2$
	C_4	28 430.00 $-284970.00\alpha_0 +366330.00\alpha_0^2$
Rod W/B=1.5	C_0	0.672 $+4.85\alpha_0 -23.93\alpha_0^2$
	C_1	25.670 $-361.90\alpha_0 +1624.00\alpha_0^2$
	C_2	-858.000 $+9512.00\alpha_0 -39580.00\alpha_0^2$
	C_3	9 219.000 $-105260.00\alpha_0 +411440.00\alpha_0^2$
	C_4	-35 145.000 $+417050.00\alpha_0 -1550300.00\alpha_0^2$
Rod W/B=2.0	C_0	0.896 $+7.24\alpha_0 -26.5\alpha_0^2$
	C_1	21.800 $-590.40\alpha_0 +2087.0\alpha_0^2$
	C_2	-1 192.000 $+17166.00\alpha_0 -58980.0\alpha_0^2$
	C_3	16 772.000 $-213330.00\alpha_0 +713640.0\alpha_0^2$
	C_4	-78 837.000 $+961870.00\alpha_0 -3146400.0\alpha_0^2$

Range: $0.18 \leq \alpha_0 \leq 0.22$, $\alpha_0 \leq \alpha \leq 0.8$

Table 4--Coefficients for Eq 7

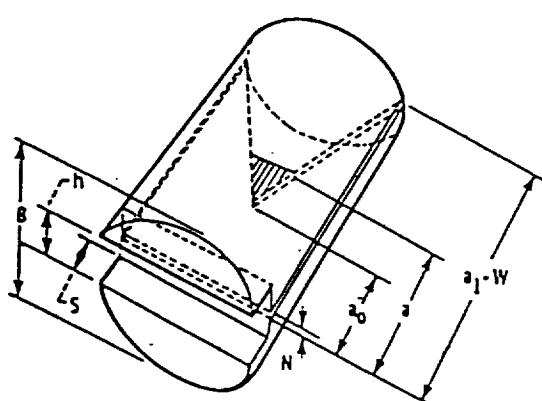
Specimen	Coeff.	Expression			
Bar W/B=1.5	D ₀	3.329	+1.026 α_0	+78.21 α_0^2	
	D ₁	-0.812	-58.080 α_0	-334.40 α_0^2	
	D ₂	-2.061	+265.260 α_0	+461.40 α_0^2	
	D ₃	4.350	-417.120 α_0	-156.10 α_0^2	
	D ₄	0.349	+219.800 α_0	-65.55 α_0^2	
Bar W/B=2.0	D ₀	4.308	-4.757 α_0	+83.77 α_0^2	
	D ₁	-6.529	-19.190 α_0	-358.70 α_0^2	
	D ₂	16.630	+172.000 α_0	+483.10 α_0^2	
	D ₃	-22.170	-313.000 α_0	-151.10 α_0^2	
	D ₄	13.220	+173.700 α_0	-72.71 α_0^2	
Rod W/B=1.5	D ₀	-2.28	+106.3 α_0	-567.0 α_0^2	+1 062 α_0^3
	D ₁	29.61	-582.5 α_0	+3 100.0 α_0^2	-5 830 α_0^3
	D ₂	-60.17	+1 167.0 α_0	-6 070.0 α_0^2	+11 589 α_0^3
	D ₃	52.60	-1 022.0 α_0	+5 051.0 α_0^2	-9 869 α_0^3
	D ₄	-15.00	+337.8 α_0	-1 517.0 α_0^2	+3 042 α_0^3
Rod W/B=2.0	D ₀	2.19	+41.95 α_0	-263.7 α_0^2	+749.0 α_0^3
	D ₁	-1.41	-128.20 α_0	+945.6 α_0^2	-3 532.0 α_0^3
	D ₂	22.30	+38.71 α_0	-691.5 α_0^2	+5 784.0 α_0^3
	D ₃	-40.95	+179.10 α_0	-703.0 α_0^2	-3 646.0 α_0^3
	D ₄	22.92	-131.30 α_0	+741.2 α_0^2	+607.4 α_0^3

Range: $0.1 \leq \alpha_0 \leq 0.35$ (bar), $0.1 \leq \alpha_0 \leq 0.40$ (rod), $\alpha_0 \leq \alpha \leq 0.8$

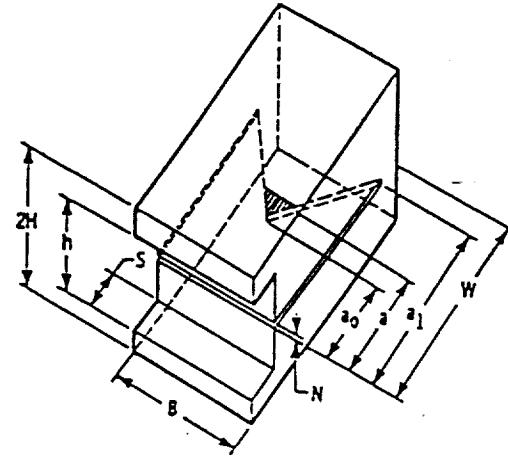
Table 5--Coefficients for Eq 8

Specimen	Coeff.	Expression		
Bar W/B=1.5	E_0	2.850	$-6.48\alpha_0$	$+61.56\alpha_0^2$
	E_1	1.177	$+26.59\alpha_0$	$-349.30\alpha_0^2$
	E_2	9.650	$-8.37\alpha_0$	$+708.90\alpha_0^2$
	E_3	-16.240	$-62.60\alpha_0$	$-597.00\alpha_0^2$
	E_4	10.450	$+56.82\alpha_0$	$+167.90\alpha_0^2$
Bar W/B=2.0	E_0	3.885	$-17.75\alpha_0$	$+94.97\alpha_0^2$
	E_1	-5.160	$+123.20\alpha_0$	$-624.20\alpha_0^2$
	E_2	34.270	$-324.50\alpha_0$	$+1562.00\alpha_0^2$
	E_3	-52.330	$+386.80\alpha_0$	$-1756.00\alpha_0^2$
	E_4	27.950	$-173.20\alpha_0$	$+741.40\alpha_0^2$
Rod W/B=1.5	E_0	3.91	$-23.18\alpha_0$	$+138.4\alpha_0^2$
	E_1	-10.01	$+237.70\alpha_0$	$-1356.0\alpha_0^2$
	E_2	51.60	$-758.80\alpha_0$	$+4284.0\alpha_0^2$
	E_3	-74.66	$+969.60\alpha_0$	$-5480.0\alpha_0^2$
	E_4	37.83	$-433.00\alpha_0$	$+2464.5\alpha_0^2$
Rod W/B=2.0	E_0	2.92	$+0.28\alpha_0$	$+26.67\alpha_0^2$
	E_1	1.68	$+28.52\alpha_0$	$-336.00\alpha_0^2$
	E_2	20.59	$-135.84\alpha_0$	$+1157.00\alpha_0^2$
	E_3	-39.16	$+218.10\alpha_0$	$-1581.00\alpha_0^2$
	E_4	22.64	$-115.78\alpha_0$	$+756.60\alpha_0^2$

Range: $0.1 \leq \alpha_0 \leq 0.35$ (bar), $0.1 \leq \alpha_0 \leq 0.40$ (rod), $\alpha_0 \leq \alpha \leq 0.8$



(a) rod



(b) bar

FIG. 1 - Chevron-notch rod and bar specimens.

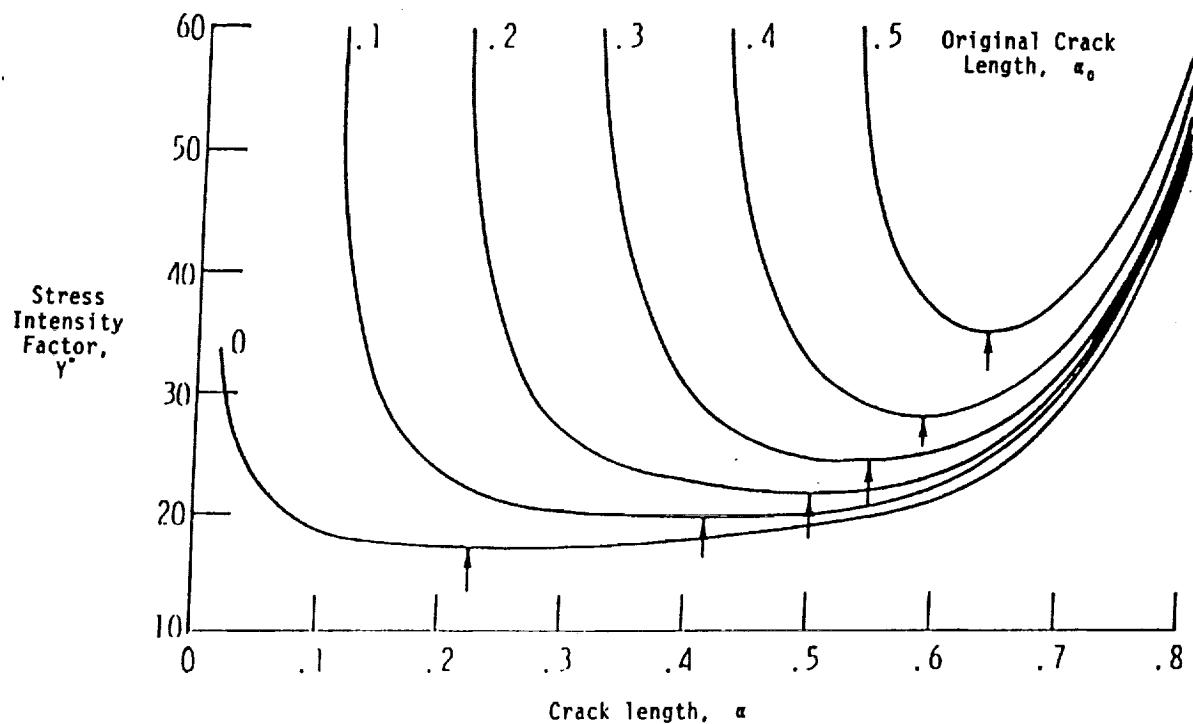


FIG. 2 - Typical variation of stress intensity factor with crack length for chevron-notch specimens [1]. Arrows denote minima.



National Aeronautics and
Space Administration

Report Documentation Page

1. Report No. NASA TM - 104367	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Analysis of Some Compliance Calibration Data for Chevron-Notch Bar and Rod Specimens		5. Report Date	
		6. Performing Organization Code	
7. Author(s) Thomas W. Orange, Raymond T. Bubsey, William S. Pierce, and John L. Shannon, Jr.		8. Performing Organization Report No. E -6167	
		10. Work Unit No. 505 - 63 - 52	
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135 - 3191		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546 - 0001		14. Sponsoring Agency Code	
15. Supplementary Notes Prepared for the Symposium on Chevron-Notch Fracture Test Experience sponsored by the American Society for Testing and Materials, Indianapolis, Indiana, May 6, 1991. Raymond T. Bubsey and William S. Pierce, retired. Responsible person, Thomas W. Orange, (216) 433-3301.			
16. Abstract This paper presents a set of equations describing certain fracture mechanics parameters for chevron-notch bar and rod specimens. They are developed by fitting earlier compliance calibration data. The difficulty in determining the minimum stress intensity coefficient and the critical crack length is discussed.			
17. Key Words (Suggested by Author(s)) Specimens; Compliance; Calibration; Stress intensity factors; Critical length; Fracture mechanics; Crack opening displacement		18. Distribution Statement Unclassified - Unlimited Subject Category 39	
19. Security Classif. (of the report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 14	22. Price* A03